GCE AS/A level

0977/01

# MATHEMATICS – FP1

## Further Pure Mathematics

A.M. WEDNESDAY, 30 January 2013

1½ hours

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

**1.** Differentiate $\dfrac{1}{2+x^2}$ from first principles. [6]

**2.** Consider the equations

$$\begin{aligned} x + 2y + 3z &= 4, \\ 2x - y + z &= 2, \\ x + 7y + 8z &= k. \end{aligned}$$

Given that these equations are consistent,

*(a)* find the value of the constant $k$, [4]

*(b)* find the general solution of the equations. [3]

**3.** The complex number $z$ and its complex conjugate $\bar{z}$ satisfy the equation

$$\mathrm{i}z + 2\bar{z} = \frac{4+6\mathrm{i}}{1+\mathrm{i}}.$$

*(a)* Determine $z$ in the form $x + \mathrm{i}y$. [6]

*(b)* Find the modulus and the argument of $z$. [2]

**4.** The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} \lambda & 1 & 1 \\ 1 & 3 & \lambda \\ 4 & 7 & 5 \end{bmatrix}.$$

*(a)* Find the values of $\lambda$ for which **A** is singular. [5]

*(b)* Given that $\lambda = 1$,

(i) determine the adjugate matrix of **A**,
(ii) determine the inverse matrix $\mathbf{A}^{-1}$. [5]

**5.** The roots of the cubic equation $x^3 + 4x^2 + 3x + 2 = 0$ are denoted by $\alpha, \beta, \gamma$.

*(a)* Show that

$$\frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha} + \frac{1}{\alpha\beta} = 2.$$ [3]

*(b)* Determine the cubic equation whose roots are $\dfrac{1}{\beta\gamma}, \dfrac{1}{\gamma\alpha}, \dfrac{1}{\alpha\beta}$. [6]

**6.** Use mathematical induction to prove that

$$\sum_{r=1}^{n} r^3 = \frac{n^2(n+1)^2}{4}$$

for all positive integers $n$. [7]

**7.** The function $f$ is defined for $x > 0$ by

$$f(x) = x^{\ln x}.$$

*(a)* Obtain an expression for $f'(x)$. [4]

*(b)* Find the coordinates of the stationary point on the graph of $f$ and determine whether it is a maximum or a minimum. [5]

**8.** The transformation $T$ in the plane consists of an anticlockwise rotation through 45° about the origin followed by a reflection in the line $x + y = 0$.

*(a)* Show that the 2 × 2 matrix representing $T$ is

$$\frac{1}{\sqrt{2}}\begin{bmatrix} -1 & -1 \\ -1 & 1 \end{bmatrix}.$$ [3]

*(b)* (i) Find the equation of the image under $T$ of the line $y = mx$.
(ii) Given that the line $y = mx$ is transformed into itself under $T$, determine the possible values of $m$. [6]

**9.** The complex numbers $z$ and $w$ are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and

$$w = z(z + 1).$$

*(a)* Show that

$$v = (2x + 1)y$$

and obtain an expression for $u$ in terms of $x$ and $y$. [3]

*(b)* The point $P$ moves along the line $y = x + 1$. Find the equation of the locus of $Q$, giving your answer in the form $v = au^2 + bu$ where $a$, $b$ are positive integers. [7]

 